Exercise 35

Find $f'(a)$.

$$f(x) = \sqrt{1-2x}$$

Solution

Determine the derivative of $f(x)$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1-2x-2h}-\sqrt{1-2x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1-2x-2h}-\sqrt{1-2x}}{h} \cdot \frac{\sqrt{1-2x-2h}+\sqrt{1-2x}}{\sqrt{1-2x-2h}+\sqrt{1-2x}} \\
&= \lim_{h\to 0} \frac{\left(\sqrt{1-2x-2h}-\sqrt{1-2x}\right)\left(\sqrt{1-2x-2h}+\sqrt{1-2x}\right)}{h\left(\sqrt{1-2x-2h}+\sqrt{1-2x}\right)} \\
&= \lim_{h\to 0} \frac{[(1-2x-2h)-(1-2x)]}{h\left(\sqrt{1-2x-2h}+\sqrt{1-2x}\right)} \\
&= \lim_{h\to 0} \frac{-2h}{h\left(\sqrt{1-2x-2h}+\sqrt{1-2x}\right)} \\
&= \lim_{h\to 0} \frac{-2}{\sqrt{1-2x-2h}+\sqrt{1-2x}} \\
&= \frac{-2}{\sqrt{1-2x}+\sqrt{1-2x}} \\
&= \frac{-2}{2\sqrt{1-2x}} \\
&= -\frac{1}{\sqrt{1-2x}}
\end{aligned}$$

Plug in $x = a$ to this formula to get $f'(a)$.

$$f'(a) = -\frac{1}{\sqrt{1-2a}}$$